Coherence Effects in Perturbations to the BCS Model Hamiltonian

$$\mathcal{H}_{pert} = \sum_{k\sigma,k'\sigma'} B_{k'\sigma',k\sigma}\, c^{+}_{k'\sigma'}\, c_{k\sigma}$$

A general perturbation scatters a single-particle from state ($k\sigma$) to state ($k'\sigma'$) with amplitude $B_{k'\sigma',k\sigma}$

In a Fermi's golden rule calculation, the transition rate will be proportional to a sum over initial and final states of

$$W_{i\to f} \sim \frac{2\pi}{\hbar}\left|\sum_{k\sigma,k'\sigma'} B_{k'\sigma',k\sigma}\right|^2 D(E_f)$$

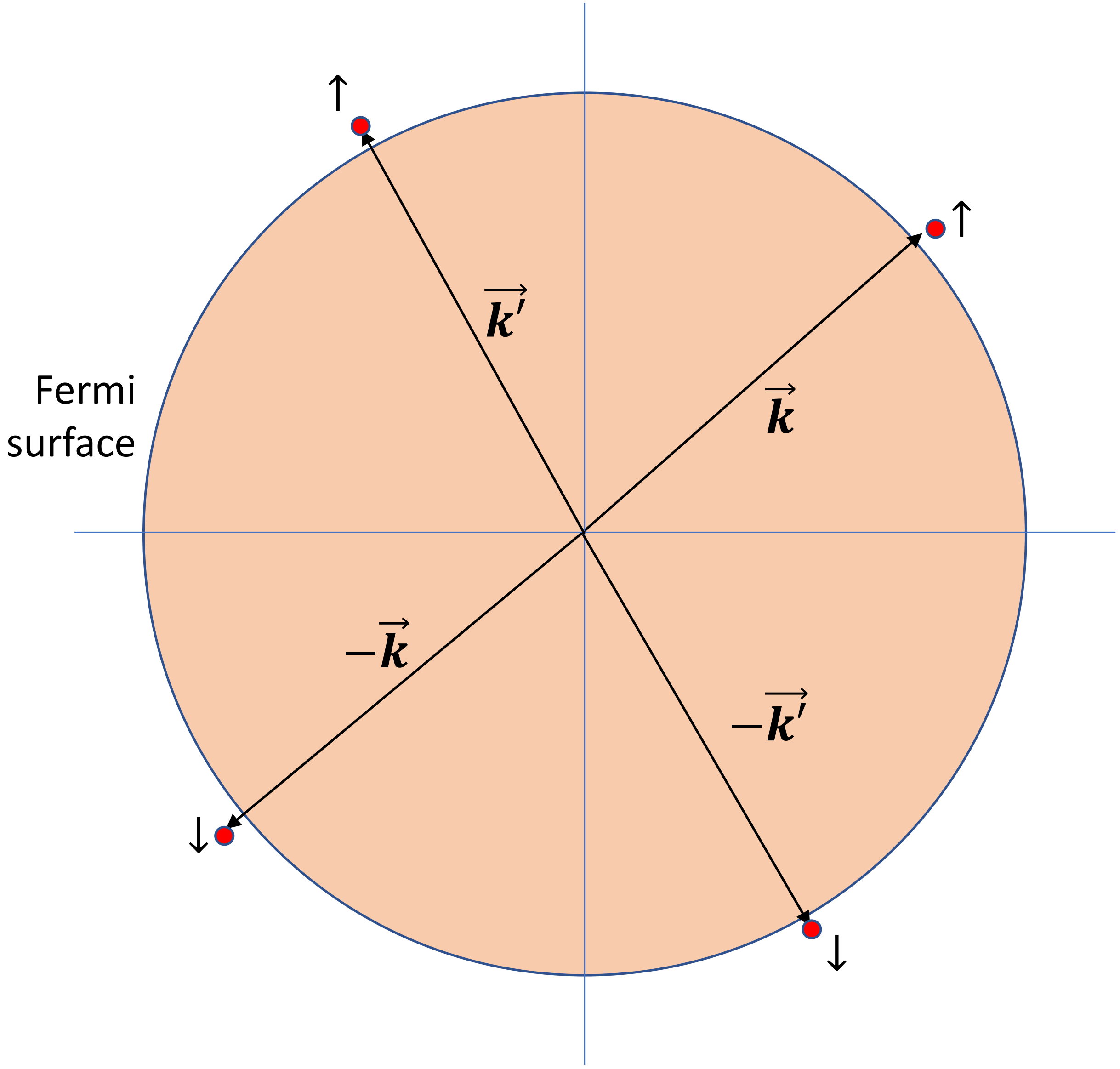
Griffiths 3rd edition
Eq. (11.81)

In a normal metal, the wavefunction is incoherent, so each term will be squared and then added

In a superconductor, the wavefunction is a coherent state of Cooper pairs, creating coherent correlations between different terms in this sum, such that those terms must be added <u>before</u> squaring and summing

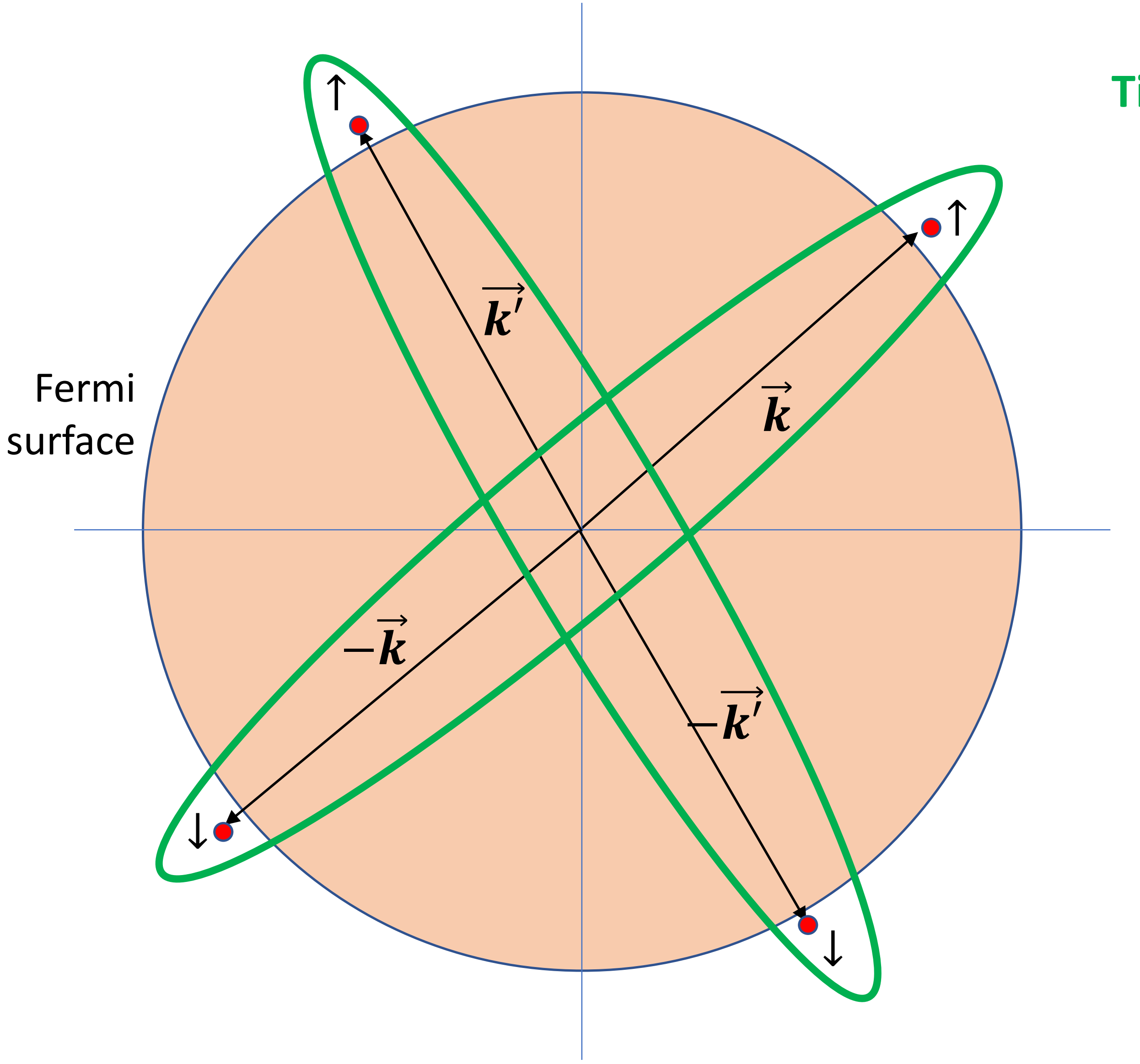
This introduces "coherence effects" in the perturbation theory of superconductors, and leads to dramatic measurable effects

Two Cooper Pairs



The perturbation acts on all particles in the system, regardless of whether they are in Cooper pairs or not

Pairing of Time-Reversed States



Time-reversed states

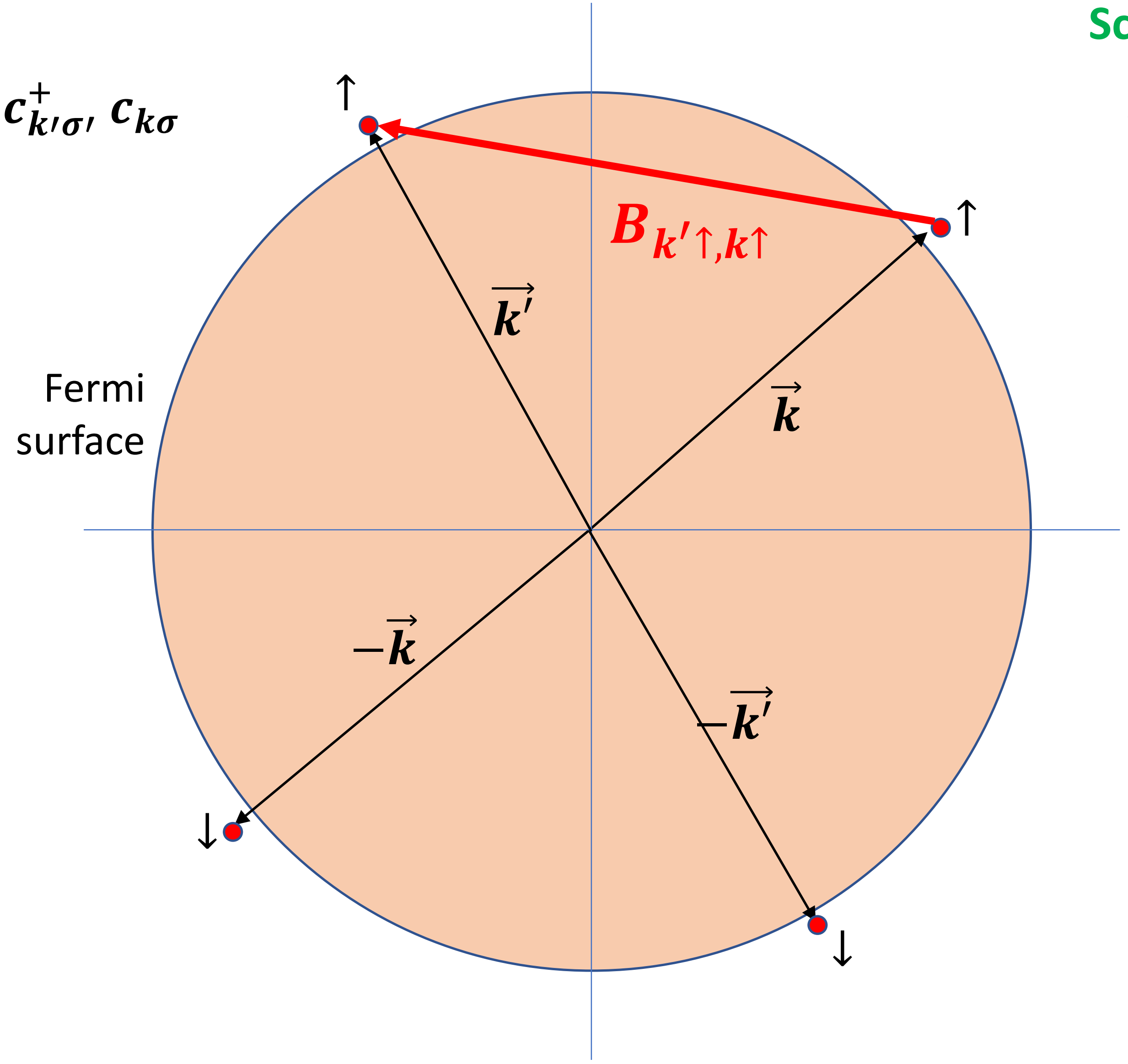
$\hat{T}\,\vec{k} \to -\vec{k}$

$\hat{T}\,\uparrow \to \downarrow$

Pairing of time-reversed states: P. W. Anderson, "Theory of Dirty Superconductors," J. Phys. Chem. Solids 11, 26-30 (1959).

Scattering Between Single-Particle States due to Perturbation (I)

$$\mathcal{H}_{pert} = \sum_{k\sigma, k'\sigma'} B_{k'\sigma', k\sigma}\, c^{+}_{k'\sigma'}\, c_{k\sigma}$$



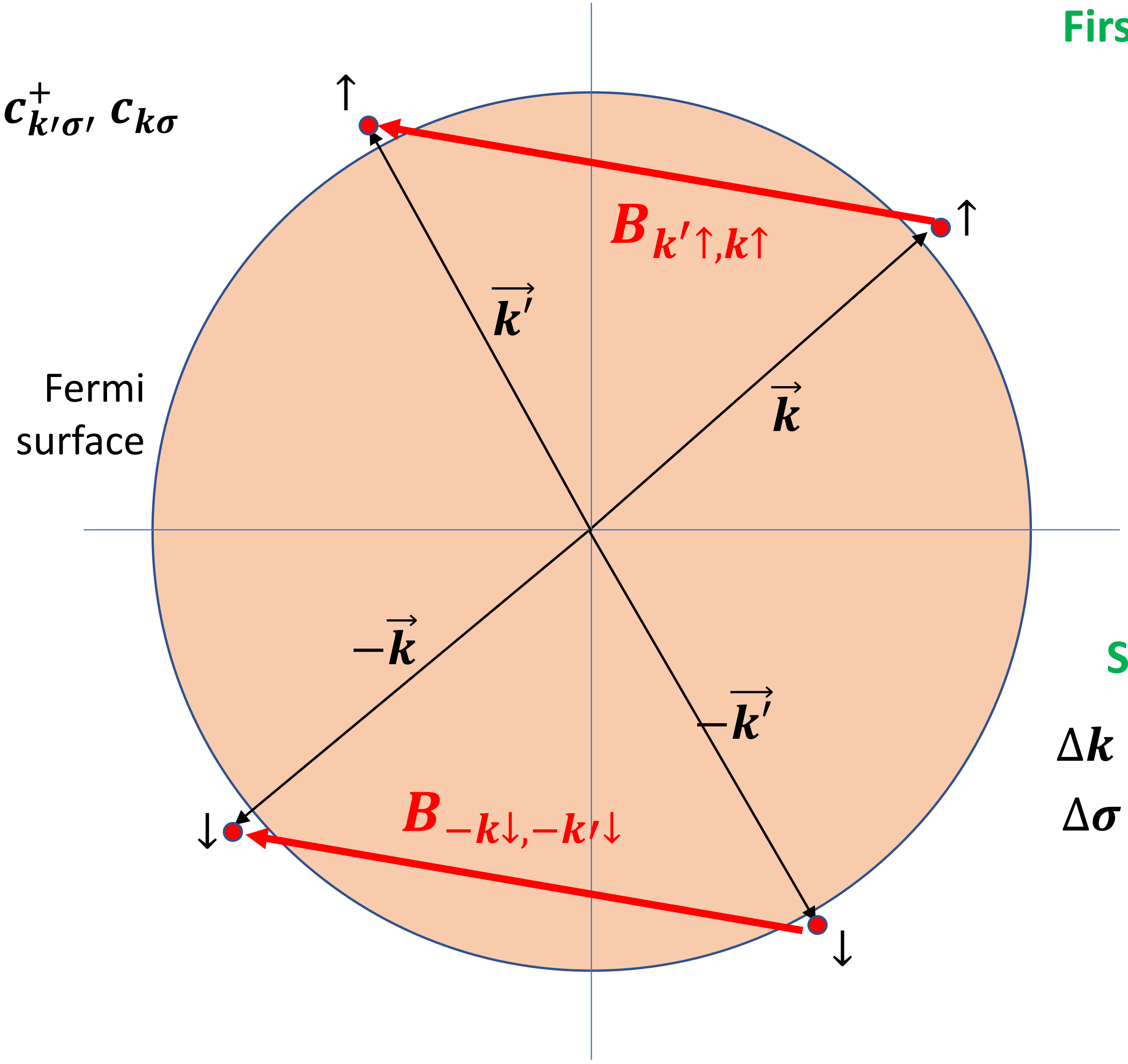
Scattering event

$$\Delta k = k' - k$$

$$\Delta \sigma = \sigma' - \sigma$$

Scattering Between Single-Particle States due to Perturbation (II)

$$\mathcal{H}_{pert} = \sum_{k\sigma,k'\sigma'} B_{k'\sigma',k\sigma}\, c^{+}_{k'\sigma'}\, c_{k\sigma}$$



First scattering event

$$\Delta \boldsymbol{k} = \boldsymbol{k}' - \boldsymbol{k}$$
$$\Delta \boldsymbol{\sigma} = \boldsymbol{\sigma}' - \boldsymbol{\sigma}$$

Consider another term in the sum …

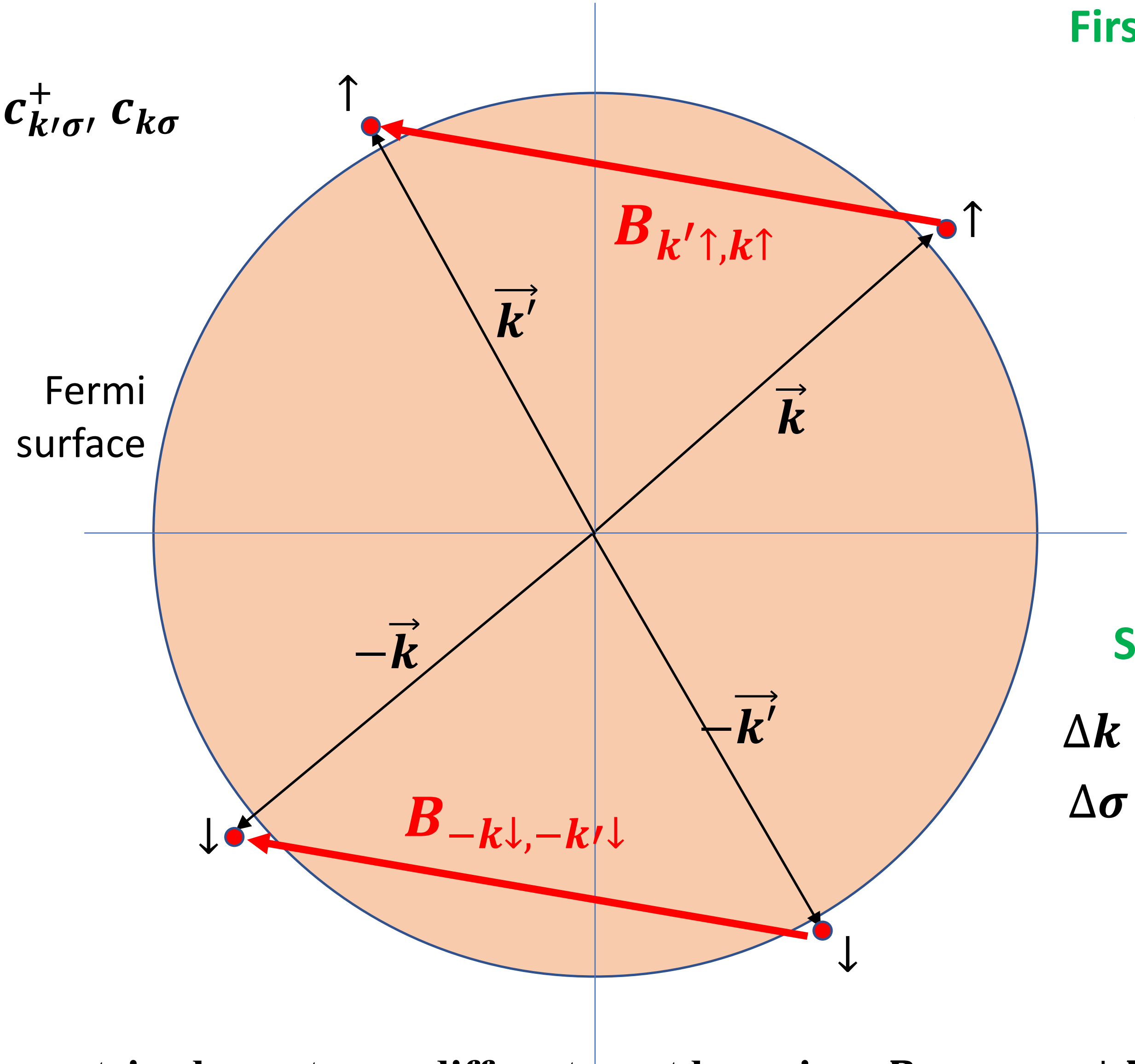
Second scattering event

$$\Delta \boldsymbol{k} = -\boldsymbol{k} - (-\boldsymbol{k}') = \boldsymbol{k}' - \boldsymbol{k}$$
$$\Delta \boldsymbol{\sigma} = -\boldsymbol{\sigma} - (-\boldsymbol{\sigma}') = \boldsymbol{\sigma}' - \boldsymbol{\sigma}$$

The <u>same momentum transfer and the same change in spin state occurs</u>, due to the correlated nature of the Cooper pairs in the original states

Scattering Between Single-Particle States due to Perturbation (III)

$$\mathcal{H}_{pert} = \sum_{k\sigma, k'\sigma'} B_{k'\sigma', k\sigma}\, c^{+}_{k'\sigma'}\, c_{k\sigma}$$



First scattering event

$$\Delta k = k' - k$$
$$\Delta\sigma = \sigma' - \sigma$$

Second scattering event

$$\Delta k = -k - (-k') = k' - k$$
$$\Delta\sigma = -\sigma - (-\sigma') = \sigma' - \sigma$$

These two matrix elements can differ at most by a sign: $B_{k'\sigma',k\sigma} = \pm B_{-k-\sigma,-k'-\sigma'}$

depending on whether the perturbation is symmetric (case I) or anti-symmetric (case II) upon time-reversal of the <u>electronic</u> state

Scattering Between Single-Particle States due to Perturbation (IV)

$$\mathcal{H}_{pert} = \sum_{k\sigma, k'\sigma'} B_{k'\sigma', k\sigma}\, c^{+}_{k'\sigma'}\, c_{k\sigma}$$

$$= B_{k'\sigma', k\sigma}\, (c^{+}_{k'\sigma'}\, c_{k\sigma} \pm c^{+}_{-k-\sigma}\, c_{-k'-\sigma'}) + \ ...$$

Case I (+): Ultrasonic attenuation, …
Case II (-): Electromagnetic absorption, nuclear spin relaxation, …

These groups of terms give rise to "coherence factors" in the calculation of absorption rates, etc.